FIITJEE

# ALL INDIA TEST SERIES

# CONCEPT RECAPITULATION TEST – I

# JEE (Advanced)-2020

## PAPER – 1

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# ANSWERS, HINTS & SOLUTIONS

## Physics

## PART – I

### SECTION – A

1. BD

Sol. Length $\propto G^x c^y h^z$

$L = [M^{-1}L^3T^{-2}]^x [LT^{-1}]^y [ML^2T^{-1}]^z$

By comparing the power of M, L and T in both sides we get

$-x + z = 0, 3x + y + 2z = 1$ and $-2x - y - z = 0$

By solving above three equations we get

$x = \frac{1}{2}, y = -\frac{3}{2}, z = \frac{1}{2}$

2. BCD

Sol. Sign between *x* and *t* is $\oplus$ hence wave is propagating in negative *x* direction.

$$\text{wave velocity} = \frac{\text{coefficient of t}}{\text{coefficient of x}} = \frac{3}{1}$$

Max value of *y* is amplitude, *y* has maximum value when denominator is least i.e. when $(x + 3t)^2$ is zero

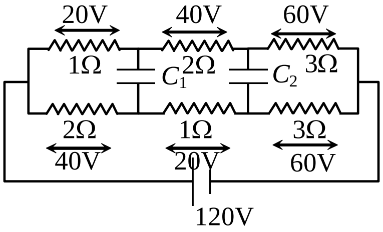
$y = 3m$

shape of pulse is given by $y = \frac{6}{2 + x^2}$ $(t = 0)$

shape of pulse is symmetrical out y-axis

3. BD

Sol. Now, potential difference across $C_1$ is 20 V and across $C_2$ is zero.
$\therefore$ charge stored in $C_1$ is 40 $\mu$C and in $C_2$ is zero.



4. BC

Sol. $v^2 = a - bx^2$
differentiating both side w.r.t. time
$$2v\frac{dv}{dt} = -2bx\left(\frac{dx}{dt}\right) \qquad \frac{dv}{dt} = A \text{ (acceleration)}$$
$$A = -bx$$
Hence motion will be S.H.M. $\omega^2 = b$, $4\pi^2 f^2 = b$, $f = \frac{\sqrt{b}}{2\pi}$
$$\text{T.E.} = \frac{1}{2}m\omega^2 (\text{Amplitude})^2 = \frac{1}{2}m\,b\,\frac{a}{b} = \frac{1}{2}ma$$

5. BD

Sol. Stopping potential does not depend on intensity and saturation current depends on intensity. Intensity I due to point source $\propto \frac{1}{r^2}$ where r is distance from the source.

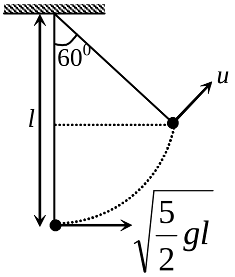
6. ABCD

Sol. $T - mg\cos 60^0 = \frac{mv^2}{l}$ … (i)

$\frac{1}{2}m\left(\sqrt{\frac{5}{2}gl}\right)^2 - mg\frac{l}{2} = \frac{1}{2}mv^2$ … (ii)

$v = \sqrt{\frac{3}{2}gl}$ and $T = 2mg$

Net force is in horizontal direction when $\theta = 60^0$



7. ABCD

Sol. $\vec{F} = 3\hat{i} - 4\hat{j}$, $W_{con.f} = U_i - U_f = 34 - 9 = 25$ & $\frac{1}{2}mv^2 - 16 = 34$

8. B

Sol. $Q_{outside}$ = earthed plate will have zero charge on outer surface for its zero potential condition.

9. B

Sol. Earthed plate will have zero charge on outer surface for its zero potential condition.

10. A

Sol. $Q_{outside} = (Q_1+Q_2)/2$, and conservation of charge.

11. C

Sol. $P = \frac{\rho RT}{M}$

12. B

Sol. VT = constant

$\Rightarrow$ $PV^2$ = constant

Thus, $\Delta W = \frac{nR(T_2 - T_1)}{1-2}$

13. D

Sol. $\Delta U = 0$

$\Rightarrow$ $\Delta T = 0$, process is isothermal.

## SECTION – B

14. 7

Sol. Given, voltage V = (100 ± 5)V

Current I = (10 ± 0.2)A

According to Ohm's law, V = IR or R = V/I

Taking log of both sides,

Log R = log V – log I

Differentiating, we get

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} - \frac{\Delta I}{I}$$

For maximum error, $\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$

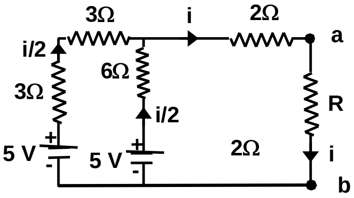
Multiplying both sides by 100 for taking percentage, we get

$$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$$

Percentage error in resistance R

$$= \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$$

$$= \frac{5}{100} \times 100 + \frac{0.2}{10} \times 100 = 7\%.$$



15. 5

Sol. $5 = \frac{i}{2}(6) + i(2) + iR$

$\Rightarrow$ $5 = 3i + 2i + iR$

$\Rightarrow$ $i = \frac{5}{5+R}$

$P = i^2R = \left(\frac{5}{5+R}\right)^2 R$

$P = i^2R$

$$\frac{dP}{dR} = 0$$
$$R = 5\Omega.$$

16. 3

Sol. $PT^2 = k$

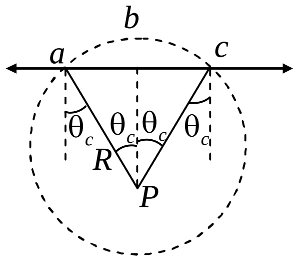
$$\gamma = \frac{1}{V}\left(\frac{dV}{dT}\right) \qquad \ldots(i)$$
$$\left(\frac{nRT}{V}\right)T^2 = k$$
$$\Rightarrow \ \frac{T^3}{V} = k'$$
$$\Rightarrow \ V = \frac{T^3}{k'} \qquad \ldots(ii)$$
$$\frac{dV}{dT} = \frac{3T^2}{K'} \qquad \ldots(iii)$$

From (i), (ii) and (iii)

$$\gamma = \left(\frac{k'}{T^3}\right)\times\left(\frac{3T^2}{k'}\right) = \frac{3}{T}$$

17. 1

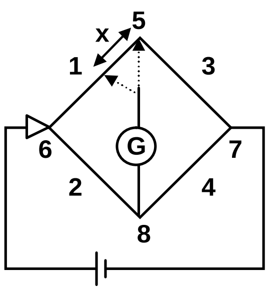
Sol. The light escape is confined within a cone of apex angle '$2\theta_c$' where $\theta_c$ is the critical angle. Imagine a sphere with source of light as its centre and the surface area abc is A.



$$\text{here } A = \int_0^{\theta_c} 2\pi R^2 \sin\theta \, d\theta = 2\pi R^2(1-\cos\theta_c)$$
$$= \pi R^2 \qquad \left[\therefore \theta_c = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ\right]$$
$$\therefore \text{ Power transfer } = P\times\frac{A}{4\pi R^2} = 4\times\frac{1}{4} = 1W$$

18. 2

Sol. For getting null point



$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$

as R2 = R4

R1 = R3

Let the pointer at point 5 is moved to left by distance x to get nul point as shown in the figure. If resistance per unit length of wire 3 is r then that of wire 1 will be 2r.

$(8 - x)2r = x \times 2r + 8 \times r$

$4x = 8$

$x = 2m$

# Chemistry PART – II

## SECTION – A

19. AD
Sol. The n-factor of $(NH_4)_2Cr_2O_7$ is 6.

20. ABD
Sol. They contain weak acid and salt of strong base.

21. AB
Sol. The overall order of the reaction is 2.

22. AB
Sol. $H_2$ and He are lighter than $CH_4$.

23. AD
Sol. $2NaI + 2H_2SO_4 \longrightarrow Na_2SO_4 + SO_2 + I_2 + 2H_2O$
KBr reacts in the same way.

24. CD
Sol. $Ni^{2+}$ contains eight and $Zn^{2+}$ contains 10 electros.
$\therefore$ $Ni^{2+} = t_{2g}^6 e_g^2$ and $Zn^{2+} = t_{2g}^6 e_g^4$

25. ACD
Sol. It has three geometrical isomers.

26. C
Sol. Coordination number of $Na^+ = Cl^- = 6$. It has fcc lattice and $Na^+$ occupies the octahedral voids.

27. C
Sol. CsCl has b.c.c unit cell. The coordination number of $Cs^+$ is 8.

28. D
Sol. Simple cubic is a primitive unit cell. It's coordination number is 6.

29. B
Sol. $2\ HCHO \xrightarrow{NaOH} HCOONa + CH_3OH$

30. C
Sol. $C_2H_5NH_2 + HNO_2 \longrightarrow C_2H_5OH + N_2 + H_2O$

31. C
Sol. $CH_3COOH + C_2H_5OH \xrightarrow{H^+} CH_3COOC_2H_5 + H_2O$ .

## SECTION – B

32. 4

Sol. $\frac{r_x}{r_{SO_2}} = \sqrt{\frac{M_{SO_2}}{M_X}} \Rightarrow \frac{4\,r_{SO_2}}{r_{SO_2}} = \sqrt{\frac{64}{M_X}}$

$\therefore M_X = 4$

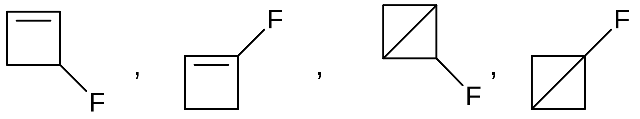
33. 6

Sol. $K_{sp} = S^2 \Rightarrow S = \sqrt{16 \times 10^{-4}} = 4 \times 10^{-2}$

$= x \times 10^{-y} \Rightarrow x + y = 4 + 2 = 6$

34. 4

Sol. The isomers are



35. 3

Sol. $C_2H_5NH_2 \xrightarrow{CH_3I} C_2H_5NHCH_3 \xrightarrow{CH_3I} C_2H_5N(CH_3)_2 \xrightarrow{CH_3I} (C_2H_5)N^+(CH_3)_2 I^-$

36. 4

Sol. The unit cell F.C.C is formed by $S^{2-}$ ions. $Zn^{2+}$ occupies half of the tetrahedral voids.

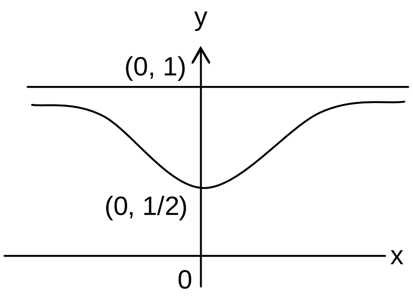
# Mathematics

## PART – III

### SECTION – A

37. BCD

Sol. Domain is $x \in R$



Also $f(x) = \left[\cos\left(\tan^{-1}(\sin\theta)\right)\right]^2$

where $\cot\theta = x$

$$= \left[\cos\left(\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)\right]^2$$

$$= (\cos\phi)^2 \text{ where } \tan\phi = \frac{1}{\sqrt{1+x^2}} = \left(\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\right)^2$$

$$f(x) = \frac{1+x^2}{2+x^2} = 1 - \frac{1}{2+x^2}$$

Range is $\left[\frac{1}{2}, 1\right); f'(x) = \frac{2x}{\left(2+x^2\right)^2}$

Hence $f'(0) = 0$

Also $\lim\limits_{x\to\infty} f(x) = 1$

Hence (B), (C), (D)

38. ABCD

Sol. We have $\alpha\left(\hat{i} - 3\hat{k}\right) + \beta\left(2\hat{i} + \hat{j}\right) + \gamma\left(\hat{i} - \hat{j} + \hat{k}\right) = -2\hat{i} - 5\hat{j} + 6\hat{k}$

On comparing, we get

$\alpha + 2\beta + \gamma = -2$ ………(i)

$\beta - \gamma = -5$ ……….(ii)

$-3\alpha + \gamma = 6$ ……….(iii)

Solving the equations (i), (ii) and (iii), we get

$\alpha = -1, \beta = -2, \gamma = 3$

39. CD

Sol. $\left(\alpha, \alpha^2\right)$ lie on the parabola $y = x^2$

$\left(\alpha, \alpha^2\right)$ must lie between the asymptotes on hyperbola

$\frac{x^2}{1} - \frac{y^2}{4} = 1$ in 1st and 2nd quadrant.

$\therefore$ Asymptotes are $y = \pm 2x$.

$\Rightarrow \quad 2\alpha < \alpha^2$

$\Rightarrow \quad \alpha < 0$ or $\alpha > 2$

and $\quad -2\alpha < \alpha^2$

$\alpha < -2$ or $\alpha > 0$

$\therefore \; \alpha \in (-\infty, -2)$ or $\alpha \in (2, \infty)$

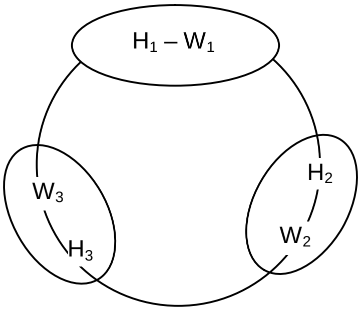
40. BC

Sol. n (All couples are together)

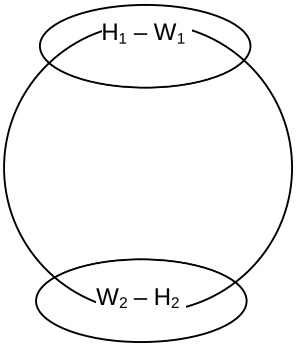
$= 2 \times 2 \times 2 \times 2 = 16$ ways

$\therefore$ $x =$ Total – 16

$x = 5! - 16 = 104$



To find y, first Let $H_1, W_1, H_2, W_2$ sit in $2 \times 2 = 4$ ways



Now $H_3$, and $W_3$ can sit in 2 ways

$\therefore y = 4 \times 2 = 8$

41. A

Sol. $\left(9 + 4\sqrt{5}\right)^n = I + f$

Let $\left(9 - 4\sqrt{5}\right)^n = f'$

$I + f + f' = \left(9 + 4\sqrt{5}\right)^n + \left(9 - 4\sqrt{5}\right)^n$

= Even integer

$0 < f + f' < 2$

$\therefore$ I is an odd integer

$\Rightarrow (a)$

42. ABCD

Sol. Now, applying $C_3 \to C_3 - g(\theta)C_1 - f(\theta)C_2$

$$\begin{vmatrix} f(\alpha) & g(\alpha) & f(\alpha + \theta) \\ f(\beta) & g(\beta) & f(\beta + \theta) \\ f(\gamma) & g(\gamma) & f(\gamma + \theta) \end{vmatrix} = 0$$

$\left[\because f(x + y) = f(x)g(y) + g(x)f(y)\right]$

43. ABC

Sol. The conditions for the existence of a solution are

1. $2\cos 2x - 1 \neq 0 \Rightarrow \cos 2x \neq \frac{1}{2} \Rightarrow 2x \neq \frac{\pi}{6}$
2. $\tan x \neq 0 \Rightarrow x \neq n\pi$
3. $\cos^2 x - 3\sin^2 x \neq 0 \Rightarrow \tan^2 x \neq \frac{1}{3} \Rightarrow x \neq \pm\frac{\pi}{6}$

As $2\cos x - 1 = 2\left(\cos^2 x - \sin^2 x\right) - 1$

$= 2\left(\cos^2 x - \sin^2 x\right) - \left(\cos^2 x + \sin^2 x\right)$

$= \cos^2 x - 3\sin^2 x$

So the given equation can be written as $b\sin x = b + \sin x$

$\Rightarrow \quad \sin x = \frac{b}{b-1}$

Which is possible if $-1 \leq \frac{b}{b-1} \leq 1$

$\Rightarrow \quad \frac{2b-1}{b-1} \geq 0$ and $\frac{1}{b-1} \leq 0$

$\Rightarrow \quad 2b - 1 \leq 0 \Rightarrow b \leq \frac{1}{2}$

But for $b = \frac{1}{2}, \sin x = -1$ which is not possible as $x \neq -\frac{\pi}{2}$.

Hence equation has a solution if $b < \frac{1}{2}$.

44. B

45. B

46. B

Sol. (for Q. 44 – 46)

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ \alpha & -1 & 3 \end{vmatrix} = 0 \Rightarrow \alpha = 4$$

Put $z = 0$ in $P_1$ and $P_2$

$2x + y = 1$ and $x - y = 2$

$\therefore \quad P \equiv (1, -1, 0)$

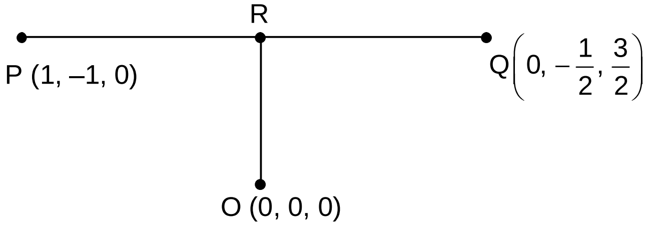
Put x = 0 in $P_1$ and $P_2$

$y + z = 1$ and $-y + z = 2$

$\therefore \; Q \equiv \left[0, -\frac{1}{2}, \frac{3}{2}\right]$

$\vec{PQ} = -\hat{i} + \frac{1}{2}\hat{j} + \frac{3}{2}\hat{k}$

Projection of PQ on x – axis $= \left|\vec{PQ}.\hat{i}\right| = 1$



Equation of line PQ is

$$\frac{x-1}{-1} = \frac{y+1}{1/2} = \frac{z}{3/2} = \lambda$$

Let $R \equiv \left(-\lambda+1, \frac{\lambda}{2}-1, \frac{3\lambda}{2}\right)$

$$\vec{OR}.\vec{PQ} = 0$$

$$\Rightarrow \; -1(-\lambda+1) + \frac{1}{2}\left(\frac{\lambda}{2}-1\right) + \frac{9\lambda}{4} = 0$$

$$\Rightarrow \quad \lambda = \frac{3}{7}$$

$$\therefore \quad R \equiv \left(\frac{4}{7}, -\frac{11}{14}, \frac{9}{14}\right)$$

$$\Rightarrow \; 7a + 14b + 14c = 2$$

$$\text{Area} = \frac{1}{2}\left[\vec{PQ} \times \vec{OQ}\right] = \sqrt{\frac{19}{16}}$$

$$\therefore \quad a - b = 3$$

47. C
48. A
49. B

Sol. (for Q. 47 – 49)

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

$$X_1 = A^{-1}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \; X_2 = A^{-1}\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}$$

$$X_3 = A^{-1}\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$$

$$\therefore \quad X = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix}$$

$$\therefore \quad \det X = 3$$

$$\det A = 1$$

$$\det(\text{adj}\, X) = 3^2 = 9$$

Sum of the elements of $X^{-1}$ is zero because $X^{-1} = \frac{1}{3}\begin{bmatrix} -1 & -2 & 0 \\ -7 & -5 & -3 \\ 9 & 6 & 3 \end{bmatrix}$

## SECTION – B

50. 3

Sol. $\alpha+\beta+\gamma=c$

$\therefore \quad \alpha\beta+\beta\gamma+\gamma\alpha=4; \alpha\beta\gamma=2$

$$\therefore \quad \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}=\frac{4}{2}=2$$

$$\because \quad f\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right)=2$$

$\Rightarrow \quad f(2)=2$

$\Rightarrow \quad 8-4c+8-2=2$

$\Rightarrow \quad c = 3$

51. 3

Sol. Since maximum value of $\cos^{-1}x$ in $[0,1]=\frac{\pi}{2}, \sum_{r=1}^{k}\cos^{-1}\beta_r=\frac{k\pi}{2}$ is possible if and only if each $\cos^{-1}\beta_r=\frac{\pi}{2}\Leftrightarrow \beta_r=0$.

$$A=\sum_{r=1}^{k}(\beta_r)^r=0$$

Thus, $\lim\limits_{x\to 0}\frac{(1+x)^{1/3}-(1-2x)^{1/4}}{x+x^2}$

$$=\lim_{x\to 0}\frac{\left(1+\frac{x^2}{3}+0\left(x^4\right)\right)-\left(1-\frac{x}{2}+0\left(x^2\right)\right)}{x(1+x)}$$

$=\lim\limits_{x\to 0}\frac{\frac{1}{2}+\frac{x}{3}+0\left(x^2\right)}{1+x}=\frac{1}{2}$. Thus the required limit is 249.

52. 7

Sol. Position vector of orthocenter is H $\left(a\hat{i}+b\hat{j}+c\hat{k}\right)$

$\overrightarrow{AH}\perp\overrightarrow{BC}\Rightarrow\left((a-1)\hat{i}+b\hat{j}+c\hat{k}\right).\left(-2\hat{j}+3\hat{k}\right)=0$

$\Rightarrow -2b+3c=0$

$\overrightarrow{BH}\perp\overrightarrow{AC}\Rightarrow\left(a\hat{i}+(b-2)\hat{j}+c\hat{k}\right).\left(-\hat{i}+3\hat{k}\right)=0$

$\Rightarrow -a+3c=0$ and $\overrightarrow{CH}\perp\overrightarrow{AB}\Rightarrow\left(a\hat{i}+b\hat{j}+(c-3)\hat{k}\right)\left(-\hat{i}+2\hat{j}\right)=0$

$\Rightarrow -a+2b=0\Rightarrow a=2b=3c=\lambda$ (say)

$$\Rightarrow H=\left(\lambda\hat{i}+\frac{\lambda}{2}\hat{j}+\frac{\lambda}{3}\hat{k}\right)$$

H lies on plane ABC

So, points A, B, C and H are collinear.

$$\therefore \begin{vmatrix} 1-0 & 0-2 & 0-0 \\ 1-0 & 0-0 & 0-3 \\ 1-\lambda & 0-\frac{\lambda}{2} & 0-\frac{\lambda}{3} \end{vmatrix} = 0$$

$$\Rightarrow 6-6\lambda-\frac{2\lambda}{3}-\frac{3\lambda}{2}=0 \Rightarrow \lambda=\frac{36}{49}$$

Hence, position vector of orthocenter is $\left(\frac{36}{49}\right)\left(i+\frac{1}{2}\hat{j}+\frac{1}{3}\hat{k}\right)$.

$$\therefore \sqrt{\frac{6}{a-b-c}} = \sqrt{\frac{6}{\frac{36}{49}-\frac{18}{49}-\frac{12}{49}}} = 7$$

53. 7

Sol. Let $P(a\cos\theta_1, b\sin\theta_1), Q(a\cos\theta_2, b\sin\theta_2)$ and $R(a\cos\theta_3, b\sin\theta_3)$ be the vertices of the triangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. The points on the auxiliary circle corresponding to these points are $P'(a\cos\theta_1, a\sin\theta_1)$, $Q'(a\cos\theta_2, a\sin\theta_2)$ and $R'(a\cos\theta_3, a\sin\theta_3)$

$\therefore\ \Delta_1 =$ Area of $\Delta PQR$

$$=\frac{1}{2}\begin{vmatrix} a\cos\theta_1 & b\sin\theta_1 & 1 \\ a\cos\theta_2 & b\sin\theta_2 & 1 \\ a\cos\theta_3 & b\sin\theta_3 & 1 \end{vmatrix}$$

$$=\frac{1}{2}ab\begin{vmatrix} \cos\theta_1 & \sin\theta_1 & 1 \\ \cos\theta_2 & \sin\theta_2 & 1 \\ \cos\theta_3 & \sin\theta_3 & 1 \end{vmatrix} \text{ and } \Delta_2 = \text{ Area of } \Delta P'Q'R'$$

$$=\frac{1}{2}\begin{vmatrix} a\cos\theta_1 & a\sin\theta_1 & 1 \\ a\cos\theta_2 & a\sin\theta_2 & 1 \\ a\cos\theta_3 & a\sin\theta_3 & 1 \end{vmatrix}$$

$$=\frac{1}{2}a^2\begin{vmatrix} \cos\theta_1 & \sin\theta_1 & 1 \\ \cos\theta_2 & \sin\theta_2 & 1 \\ \cos\theta_3 & \sin\theta_3 & 1 \end{vmatrix}$$

Clearly, $\frac{\Delta_1}{\Delta_2}=\frac{1}{7} \Rightarrow \frac{\Delta_2}{\Delta_1}=7$

54. 9

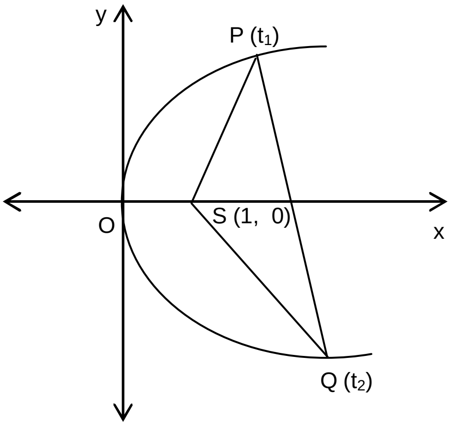
Sol. We have $3h=\left(t_1{}^2+t_2{}^2+1\right)$ ………..(1)

$3k=2(t_1+t_2)$ ………..(2)

and $t_2=-t_1-\frac{2}{t_1}$ ………..(3)

$$\Rightarrow t_1 + t_2 = \frac{-2}{t_1}$$

Eliminate $t_1$ and $t_2$ to get



$$\therefore \ \frac{3k}{2} = \frac{-2}{t_1} \Rightarrow t_1 = \frac{-4}{3k}$$

$$t_2 = \frac{3k}{2} + \frac{4}{3k}$$

Substitute the values of $t_1$ and $t_2$ in (1) we get

$$3h = \frac{16}{9k^2} + \left(\frac{3k}{2} + \frac{4}{3k}\right)^2 + 1$$

$$3h = \frac{32}{9k^2} + \frac{9k^2}{4} + 5$$

$$(3x - 5)36y^2 = 128 + 81y^4$$

Hence $g = 36; k = 81$ and $w = 128$